

# Utilizing Graph Theory for the Smooth Flow of Transportation in Areas

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## Abstract

Mathematical Science is the only study that is utilized in numerous of other sciences to make the outcomes. Whether it's about chemicals or motion of a body, computer or real life problems mathematical techniques are widely in use to solve these problems. Mathematics is not just all about the numbers. It has been utilized to solve many real life applications. Transportation is one of them which is the main objective of this research. Making a perfect transportation plan is important for a country's esteem. It shows discipline and progress of a nation. Graph theory hypothesis will assists in making a better plan of the transportation. The main purpose of this paper is to apply minimum spanning tree problem and Kruskal's algorithm to make a smooth flow of transportation by applying it in the blitz areas. This will help to find the shortest path between the places, hospitals are the main focus.

*Keywords:* Graph Theory; Euclidian Graph; Spanning Tree algorithm; Kruskal's Algorithm.

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## Introduction

Graph theory is a growing field of mathematics with a wide range of applications. A graphical representation of a collection of things connected by routes/edges is an extension towards advance mathematics; in mathematics, the components and paths are referred to as vertices and edges [3]. Graph theory is a relatively new field of mathematics and has become a very important component in many applications in real-world problems such as transportation networks, computer, science, social networks etc. In 1990's, graph theory was mostly used to transportation systems in relation to economics, most notably in an effort to predict the effects of Pakistan city Lahore and various freeway built in cities [1].

An ordered pair  $G = (V, E)$  denotes a graph, where  $V(G)$  is the set of vertices and  $E(G)$  is the set of all the edges. Various processes i.e.; computer science, biology, physics, and information systems may be modeled using graphs. [6] When Swiss mathematician Leonhard Euler tried to solve the seven bridges of Königsberg issue in 1735, the history of graph theory officially began. It was an ancient issue when the question (see Figure 1(a)) arises. Is it feasible to pick a starting place in the city and map out a walking path that crosses every bridge specifically once? Each of the four landmasses is depicted on this simplified map by Euler as a single point, known as a vertices, with lines connecting them, known as edges, to symbolize bridges.

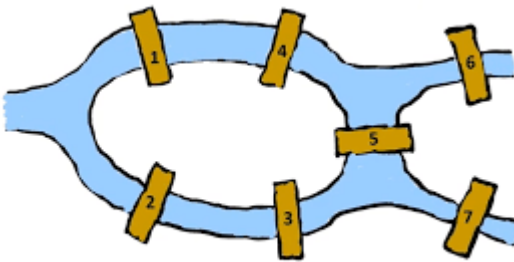


Figure 1(a)

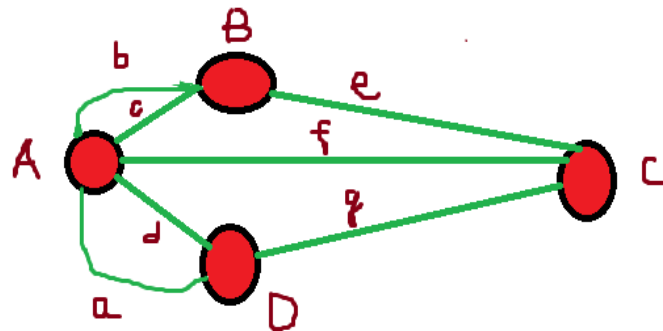


Figure 1(b)

And we can simply count the degrees of each vertex due to this simplified graph (Figure 1(b)). In his initial graph theory article, Euler demonstrated that there are no such pathways in this particular situation. He then established a method for identifying the edges of a Euclidian graph [2, 4, 5].

Graph theory was first used to road transportation systems in the late 1950s and continued into the 1970s. Avondo Bodino was the one who develop an application of graph theory to transport networks (Citation 1962).

It wasn't until the 1980s and 1990s that graph theory principles were applied to the study of public transportation networks. The existence of several public transportation modes might make it difficult to examine transit networks properly; after all, how can the edges of streetcars, buses, light rail systems, and metros be compared?

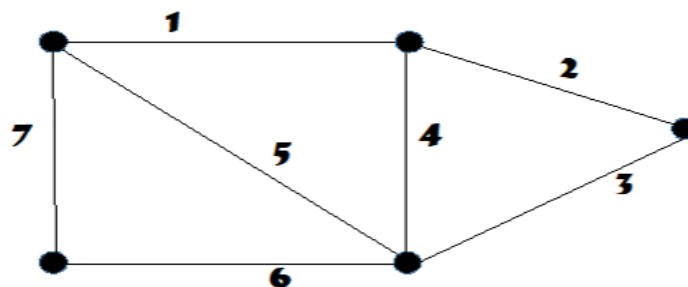
In addition, the purpose of transit is to get people from one place to another while minimizing travel time, avoiding needless transfers, and maintaining a number of other important attributes including convenience, dependability, and safety. As a result, there are a lot more factors to take into account while researching transit networks. Furthermore, some characteristics of public transportation networks (such as the existence of lines, their overlap, etc).

A transportation network provides a means for goods and citizens to travel from a starting location to a destination location. Each of these networks plays an essential role in civilization. Bus networks and bus stations are essential parts of our national infrastructure and are connected by complex systems. Any situation with linked elements can be symbolized by graph theory. In this research paper, I will use graph theory algorithms and theorems that will help to improve potential vulnerabilities in infrastructure networks. This article presents various applications of graph theory that can be used to optimize and analyze locations for transportation systems. In a transportation graph, each edge is associated with a weight or cost, which can represent factors such as travel time, distance, congestion levels, or monetary cost. These weights can be dynamic, changing with real-time traffic conditions or other variables. One of the primary applications of graph theory in transportation is route optimization. Algorithms like Kruskal's algorithm and Prim algorithm can find the shortest or most efficient paths between two nodes, taking edge weights and other constraints into account. This is essential for GPS navigation systems and route planning tools. "Kruskal's greedy algorithm finds a minimum spanning tree for a weighted, undirected graph".

## FINDING MINIMUM WEIGHT SPANNING TREE

### (KRUSKAL'S ALGORITHM)

A graph is a tree if and only if it is connected and is acyclic. If the tree has minimum weight then it is said to be a minimum spanning tree. A minimum spanning tree has  $V-1$  edges, where  $V$  is the number of vertices in the original graph.



**Undirected  
Graph**



**Spanning  
Tree  
Cost=17**

**Minimum Spanning  
Tree  
Cost = 12**

In the city, there are many roads that lead to a hospital, so the city government is supposed to set the shortest route for the vehicles to reach the hospital in a minimum time. The total weight of edges makes the cost of the spanning tree. There are many spanning trees in a graph. If the cost of the spanning tree is minimum among all the others spanning tree then such a spanning tree would be called minimum spanning tree.

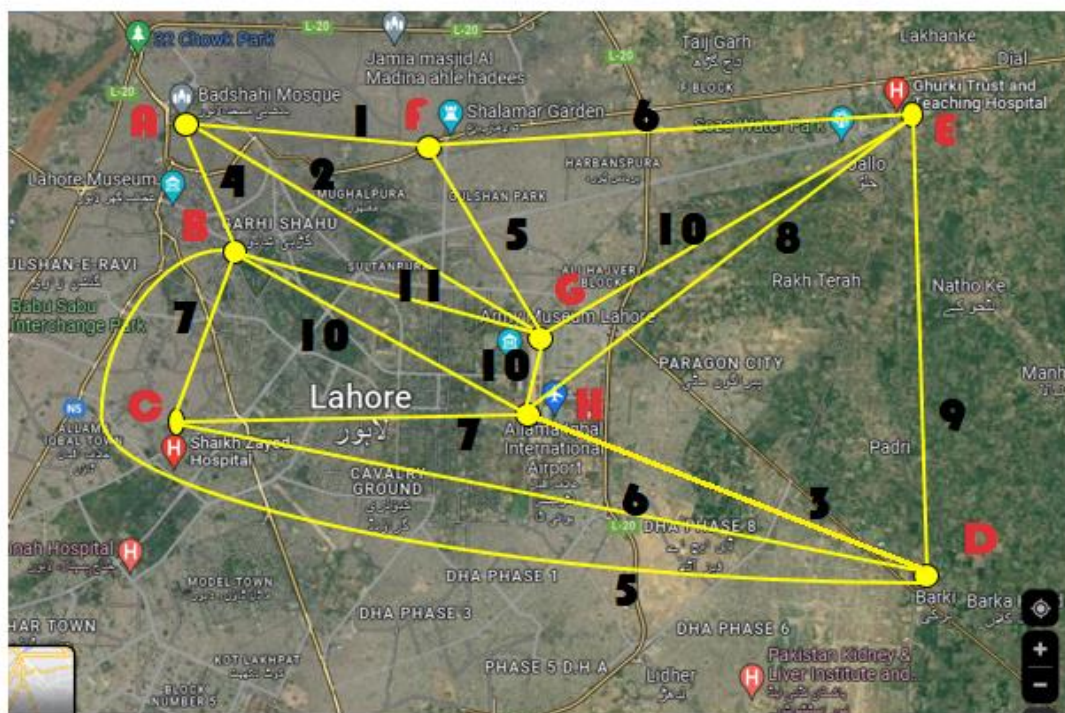
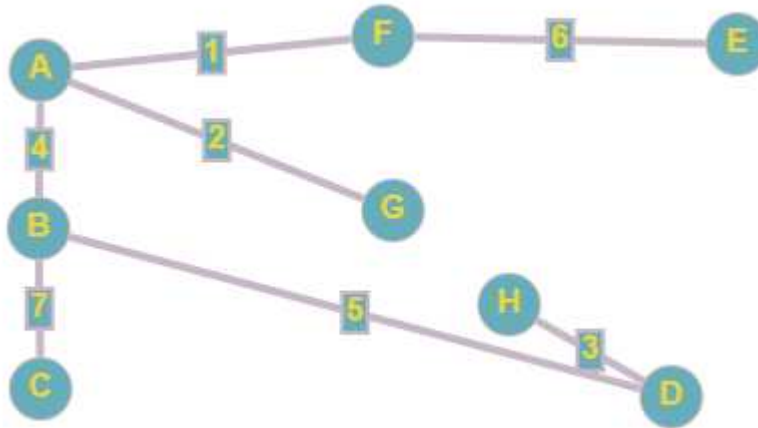


Figure 2 Original Graph

Figure 2 is the map of Lahore city. By using the map, we will calculate the shortest distance of Sheikh Zaid and Ghurki trust hospital from the some of the rushy areas by using Kruskal's algorithm. These areas include Allama Iqbal international hospital, Garhi Shahu, Army Museum, Shalimar Garden, Badshahi Mosque and Barki which are marked on the map with yellow vertices.



Kruskal's algorithm is the one through which a minimum spanning tree can be built by adding minimum weight edges. Kruskal's algorithm produces a minimum spanning tree in a connected weighted graph. To make a spanning tree with the algorithm, firstly we select an edge of minimum weight from the graph, and then from the remaining graph we will select another edge of minimum weight. For the third edge, choose any remaining edge of minimum.

|   | A | B | C | D | E | F | G  | H  |
|---|---|---|---|---|---|---|----|----|
| A | - | 4 | * | * | * | 1 | 2  | *  |
| B |   | - | 7 | 5 | * | * | 11 | 10 |
| C |   |   | - | 6 | * | * | *  | 7  |
| D |   |   |   | - | 9 | * | *  | 3  |
| E |   |   |   |   | - | 6 | 10 | 8  |
| F |   |   |   |   |   | - | 5  | *  |
| G |   |   |   |   |   |   | -  | 10 |
| H |   |   |   |   |   |   |    | -  |

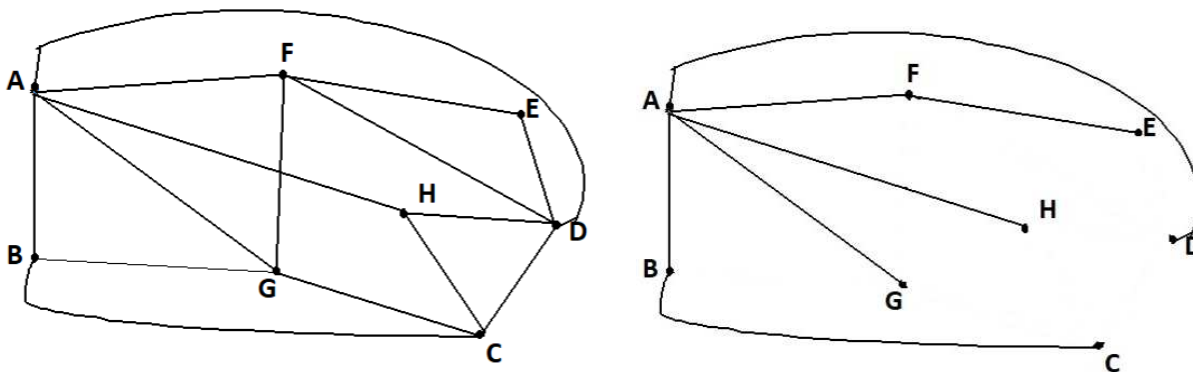
The minimum cost of the above graph is  $1+3+4+5+5+6+7=31$  billion which is the minimum cost. Figure shows the graph when we used Kruskal's algorithm.

### SPANNING TREE ALGORITHM

Graph theory plays a vital role in the optimization of transportation network by using spanning tree algorithm. Identifying minimum spanning tree within a graph is particularly helpful, when dealing with transportation problem. Whether its road, rail or interconnected system, with the help of the algorithm we can find the efficient and cost-effective route that can connects all the roads with in the network.

By making spanning trees the government can reduce the cost of transportation infrastructure by eliminating redundant links and create a simple, acyclic structure. It will help us to minimize the use of resources and traffic management become easier.

Moreover, spanning tree can help us identifying the critical routes, and we can take help by making bridges over these cluster roads to divide the traffic and make the transportation work more efficiently. Thus, spanning tree serve as the fundamental tool of graph theory in transportation management and robust network.



### Conclusion

This study presents a new approach to the optimal solution of the transportation problem using the spanning tree and Kruskal algorithm. Various techniques have been developed in the literature to solve the transportation problem, but this approach plays an important role in transportation and graphs. This innovative

approach consumes less computer time and minimal steps to find the optimal solution to the transportation problem compared to existing methods. However, this new method is based on the distribution of transportation costs in a transportation matrix and can be applied to all balance and equilibrium problems using more variables. Therefore, this method may be of interest for future work on real topological and graphical transportation problems.

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