Cumulative Sum Modifications Control Chart in Monitoring Mean Process, Study of Case Laying Hens Training Coop at SMK PP N Sembawa

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Abstract

A control chart is one of the tools of Statistical Process Control (SPC) used to monitor a production process. The control chart's ability to detect data that crosses the border is often done using variable transformation. In this paper, we will introduce a cumulative sum control chart (CUSUM) based on a Link Relative transformation technique using a regression approach. The results showed that the Link Relative control chart with regression was more sensitive compared to the previous CUSUM control chart and Link Relative CUSUM control chart. The results will be applied to the determination of the limit Feed Conversion Ratio (FCR) in PPN SMK Sembawa Palembang, Indonesia.

Keywords: control chart, cumulative sum, link relative, regression approach;

1. Introduction

The management activities in the breeding of chickens and eggs, if done well, then the brewing of chicken and egg that is done can be successful. Production metrics, including as feed consumption, egg production, mortality, feed conversion, and egg abnormalities, can be used to gauge the performance of poultry farming operations. Poultry breeding is one of the agricultural projects done at the enterprise unit of the SMK PP Negeri Sembawa (SMK PP N Sembawa).

The feed conversion value, also known as the feed conversion ratio, or FCR, is typically employed as a measure of the breeding success of chickens. The FCR in turkeys refers to the conversion ratio of the feed used by the turkey to produce eggs. The lower the FCR value, the more efficient the use of feed, which means producers can better results with fewer feed amounts. Based on the enterprise unit's activity at SMK PPN Sembawa, the limit FCR will be determined on this paper using statistical process control (SPC).

SPC is a method to control a production process's quality. SPC involves collecting data, using statistical analysis to determine mean and variability in the process of production, and then taking necessary steps to guarantee that the product is produced in accordance with the necessary specifications. The primary purpose of SPC is to find mean deviations or undesired variations in the process of production before they reach unacceptable levels. The control chart is among the most commonly used statistical methods. (Control charts). Upper control limit (UCL), center limit (CL), and low control limit (LCL) are the three control chart parameters.

Shewhart's control chart consists of a center line that represents the mean values of the study variables, the upper control limit (UCL) and the lower control limit (LCL). The procedure is considered to be statistically controlled at this stage based on the data point point that is in these lines. Any point that crosses any of these two lines denotes an out-of-control process. Shewhart control charts were introduced in 1920, however, for small data changes with a change of less than 1.5σ, researchers suggested the use of EWMA and CUSUM control charts, Montgomery (2009).

The CUSUM control chart was first introduced by Page in 1954. This control chart describes the amount of cumulative deviations of the sample value of a characteristic quality of the target value over time. Several authors like Duncan in 1974, Lucas in 1976, Hawkins in 1981, Lucas and Saccucci in 1982, 1990 stated that the CUSUM control chart is much more efficient than the conventional x control chart to detect smaller variations within the average, Koshti (2011). Faisal et.all (2018) states that there are some sensitivity rules that indicate random or irregular observation patterns. Therefore, any systematic pattern that appears on the control chart will indicate that the process is out of control, so corrective action must be taken to stabilize the process. This paper proposes a link relative, where the Link Relative transforms the original variable into a relative form to its average. In this paper, it will be proposed the plot of the Link Relative control CUSUM proposed by the Faisal with the estimation of the parameters in the formation done by simple regression method.

Subsequently obtained control charts will be used to statistically determine the control limit of the FCR using the Cumulative Sum Control Chart (CUSUM), and the Link Relative CUSUM and the Link Relative CUSUM control chart based on the regression method

2. Research

2.1. CUSUM Control Chart

The cumulative sum control chart (CUSUM) was initially introduced by Page in 1954. The CUSUM control chart is suitable for detecting small and continuous shifts in a process. The cumulative deviation from the target value is used in the CUSUM control chart. The V-mask CUSUM and the tabular CUSUM are the two forms of the CUSUM control chart that are used for evaluating out-of-control conditions however the table CUSUM is the one that is most commonly used. (Montgomery, 2009).

A tabular CUSUM that is one of the presentations for a control chart is used to accumulate shifts above the target values represented by $C_i^+$and $C_i^-$for shifts below the target value. (Montgomery, 2009). The values of $C_i^+$and C_i^-can be defined from:

$$C_i^+ = max[0,\ x_i - (\mu_0 + K) + C_{i-1}^+] \tag{1}$$

$$C_i^- = min[0,\ x_i - (\mu_0 - K) + C_{i-1}^-] \tag{2}$$

where:

C_i^+ : i^{th} upper CUSUM

C_i^- : i^{th} lower CUSUM

x_i : i^{th} observation

μ_0 : target value

$C_0^+ = C_0^- = 0$: initial value

The layout of the CUSUM control chart requires two parameters namely K and H. The K value is a slack value determined from:

$$K = k\sigma \tag{3}$$

Some previous studies have suggested a value of k is 0.5 or k is 0.4 (Montgomery, 2009). Whereas H is the decision interval or control limit, that is:

$$UCL = h\sigma \tag{4}$$

$$LCL = -h\sigma \tag{5}$$

using a value of h=4 or h=5 generally will give a control chart CUSUM that has good ARL properties against a shift of about 1σ in the average process.

2.2. Link Relative CUSUM Control Chart

Each data point is distributed as a cumulative sum of the relative difference between the actual data point and the intended middle value on a CUSUM control chart with a link relative. Consider Wi~N(μ_W,σ_W), with μ_W and σ_W assumed known and positive, so the link relative variable can be defined as:

$$Y_i = \begin{Bmatrix} \frac{W_i}{\mu_W} \; if \; W_i > \mu_W \\ -\frac{\mu_W}{W_i} \; if \; W_i < \mu_W \end{Bmatrix} where \; i = 1,2,3, \ldots \tag{6}$$

where i denotes the sample number. Yi compares each observation with the process mean. It is computed from original process observations and is also dependent on process mean. So actually, it is a variable that is designed to show the direction of observations relative to the process mean. It is interesting to note that (1) $\lim \; E(Y) = 0$; (2) $\sigma_Y = \sigma_W$ as $\mu_W \to \infty$

Further, using Yi we define a transformed variable $\widehat{W}_i$, in a more target concentrated form as

$$\widehat{W}_i = a + bY_i \tag{7}$$

where we assume $a = \mu_W$ W for the sake of optimality and for the same concern b is given as $b = \sqrt{\frac{2}{\pi}}\sigma_W$.

Mean and standard deviation of the model presented earlier are represented as μ_W dan σ_W. The proposed Link Relative CUSUM control chart has the following plot statistics:

$$C_i^+ = max[0,\ \widehat{W}_i - \mu_{\widehat{W}} - k\sigma_{\widehat{W}} + C_{i-1}^+] \tag{8}$$

$$C_i^- = max[0,\ \widehat{W}_i - \mu_{\widehat{W}} + k\sigma_{\widehat{W}} + C_{i-1}^-] \tag{9}$$

where $C_0^+ = C_0^- = 0$ are the starting values. The parameter $H = h\sigma_{\widehat{W}}$ is the decision interval or limit control

$$UCL = h\sigma_{\widehat{W}} \tag{10}$$

$$LCL = -h\sigma_{\widehat{W}} \tag{11}$$

2.3. Link Relative CUSUM Control Chart with Regression Approach

The principle of the Link Relative CUSUM control chart with this regression is to perform data transformation using regression, that is, by modeling the relationship between dependent and independent variables using the regression method, then applying transformation to those variables to meet regression assumptions or improve the quality of the model. When regression assumptions like linearity, normality, or homoscedasticity are violated, such data manipulation is typically required.

From the equation (7) is: $\widehat{W}_i = a + bY_i$, then assuming $y_i = \widehat{W}_i$ and $x_i = Y_i$, , so that it was obtained:

$$\hat{y} = a_R + b_R x \tag{12}$$

a_R and b_R values can be obtained with the following formula:

$$a_R = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2} \tag{13}$$

$$b_R = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} \tag{14}$$

Furthermore, using the values of a_R, b_R and Yi, to define the $\widehat{W}_{Ri}$ transformation variable, the following equation is obtained:

$$\widehat{W}_{Ri} = a_R + b_R Y_i \tag{15}$$

Then a Link Relative CUSUM control chart can be made with the proposed Regression the following plot statistics:

$$C_i^+ = max[0,\ \widehat{W}_{Ri} - \mu_{\widehat{W}_R} - k\sigma_{\widehat{W}_R} + C_{i-1}^+] \tag{16}$$

$$C_i^- = max\left[0,\ \widehat{W}_{Ri} - \mu_{\widehat{W}_R} + k\sigma_{\widehat{W}_R} + C_{i-1}^-\right] \tag{17}$$

where $C_0^+ = C_0^- = 0$ are the starting values. is the decision $H = h\sigma_{\widehat{W}_R}$ interval or limit control.

$$UCL = h\sigma_{\widehat{W}_R} \tag{18}$$

$$LCL = -h\sigma_{\widehat{W}_R} \tag{19}$$

3. Study of Case

The data collection location for this paper is located in Banyuasin Regency, South Sumatera Province, namely in the Laying Hen Training Coop. The data used in this study is secondary data, namely data on the FCR value of laying hens during the production period, from 30 to 80 weeks of age. The data variable used in this study is (x) is the FCR value of laying hens. Table 1 is the FCR data of laying hens observed from the 30th to the 80th week as preliminary data. This preliminary data is used as the basis for determining the control chart that will serve as the next data control.

Table 1. FCR Value Data f or Laying Hens

No.	Week	Mean	No.	Week	Mean	No.	Week	Mean
1	30	1.98	18	47	1.92	35	64	1.94
2	31	1.95	19	48	1.93	36	65	1.92
3	32	2.03	20	49	1.93	37	66	1.94
4	33	2.13	21	50	1.91	38	67	1.97
5	34	1.97	22	51	1.91	39	68	1.82
6	35	2.00	23	52	1.94	40	69	1.81
7	36	1.99	24	53	1.96	41	70	1.86
8	37	1.98	25	54	1.90	42	71	1.91
9	38	1.96	26	55	1.92	43	72	1.96
10	39	2.00	27	56	1.90	44	73	1.90
11	40	1.98	28	57	1.91	45	74	1.90
12	41	1.95	29	58	1.95	46	75	1.87
13	42	1.94	30	59	1.97	47	76	1.95
14	43	1.93	31	60	1.95	48	77	1.94
15	44	1.92	32	61	1.94	49	78	1.99
16	45	1.91	33	62	1.99	50	79	1.89
17	46	1.90	34	63	1.89	51	80	1.94

CUSUM control chart using data in Table 1, based on equations (1), (2), (4) and (5) the following CUSUM control chart data will be obtained:

UCL = 0,2
LCL = -0,2

When all the sample data is displayed together with the control chart obtained, the results can be seen in Figure 1.

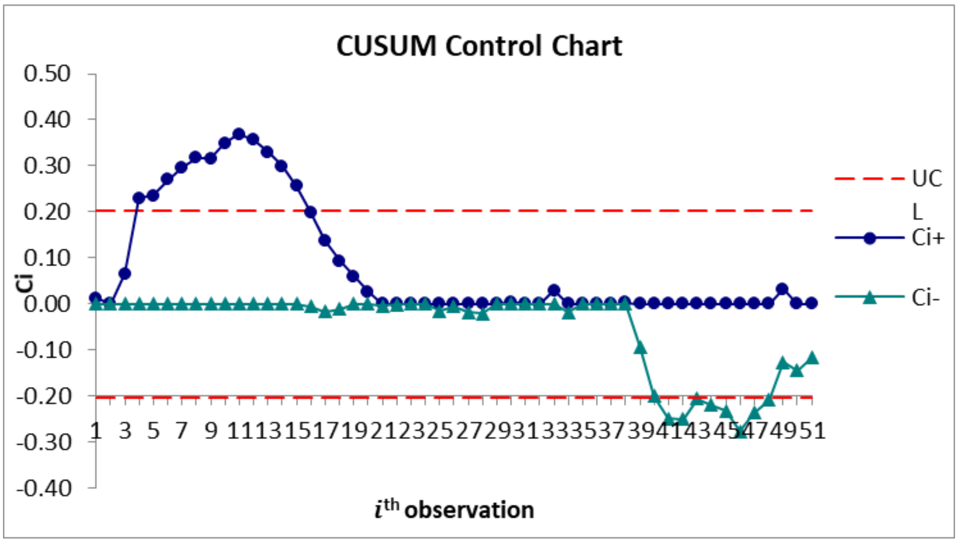

Fig. 1. CUSUM Control Chart

Figure 1 shows that some data are still outside the control limits, there are 22 out of 51 data which are used as preliminary data. In this early stage a control chart will be determined in which all data is within the control limits. If obtained later, the control chart can be used in monitoring further data. After 22 data is removed, a new custom control chart will be obtained as follows:

UCL = 0,16

LCL = -0,16

With a control chart and sample data as shown in Figure 2.

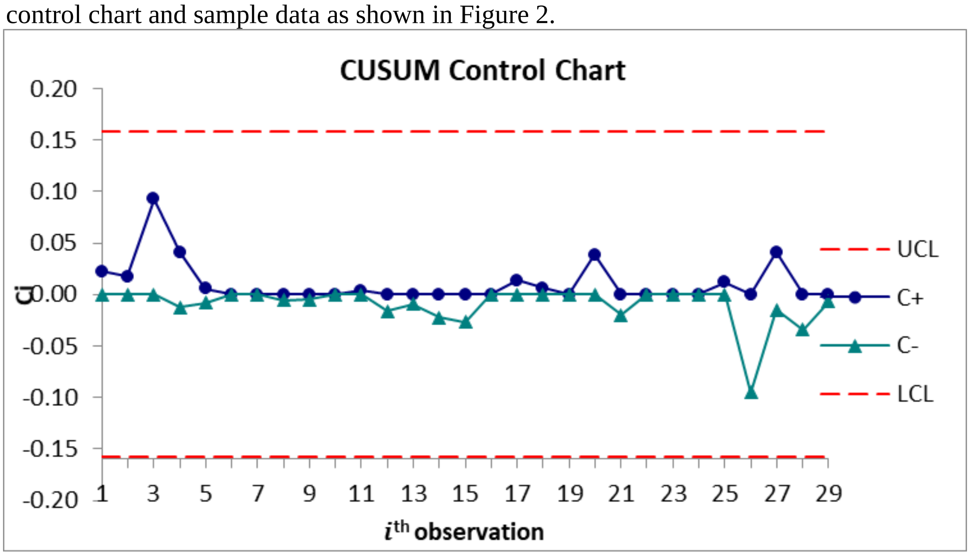

Fig. 2. CUSUM Control Chart after the out-of-control data has been removed

In Figure 2, it can be seen that all data is within the control limits, so this control chart can be used to monitor further data. Link Relative CUSUM control chart using the data in Table 1, based on equations (8), (9), (10) and (11) the CUSUM control chart data will be obtained as follows:

UCL = 0,167

LCL = -0,167

When all the sample data is displayed together with the control chart obtained, the results can be seen in Figure 3.

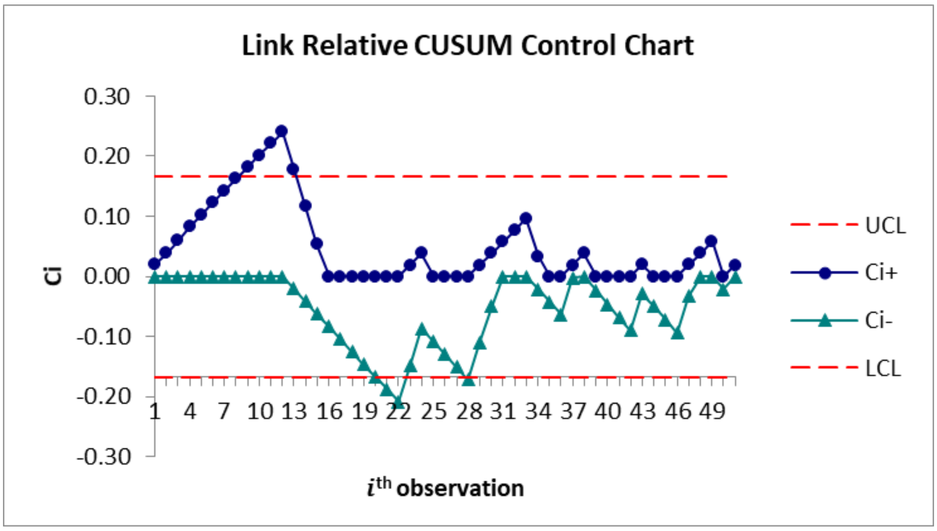

Fig. 2. Link Relative CUSUM Control Chart

Figure 3, shows some data are still outside the control limits, there are 8 data out of 51 data which are used as preliminary data. In this early stage a control chart will be determined in which all data is within the control limits. The Control Chart obtained next, can be used in monitoring further data. After 8 data is removed, a new custom control chart will be obtained as follows:

UCL = 0,167

LCL = -0,167

With a control chart and sample data as shown in Figure 4

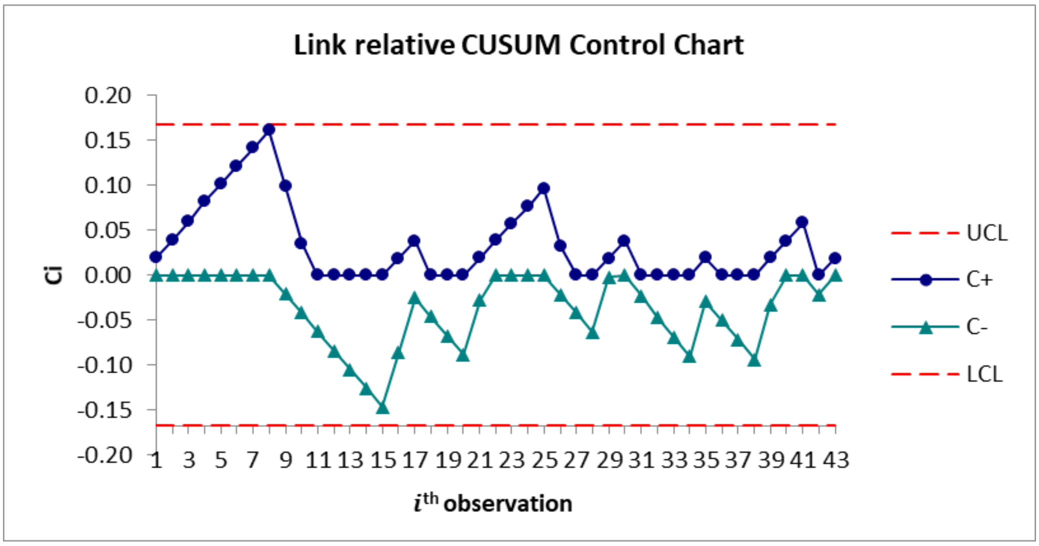

Fig. 4. Link Relative CUSUM Control Chart after the out-of-control data has been removed

In Figure 4, it can be seen that all data is within the control limits, so this control chart can be used to monitor further data. Control chart Relative CUSUM link Regression method using the data in Table 1, based on equations (16), (17), (18) and (19) the data link retalive CUSUM control chart for the regression method will be obtained as follows:

UCL = 0,167

LCL = -0,167

When all the sample data is displayed together with the control chart obtained, the results can be seen in Figure 5

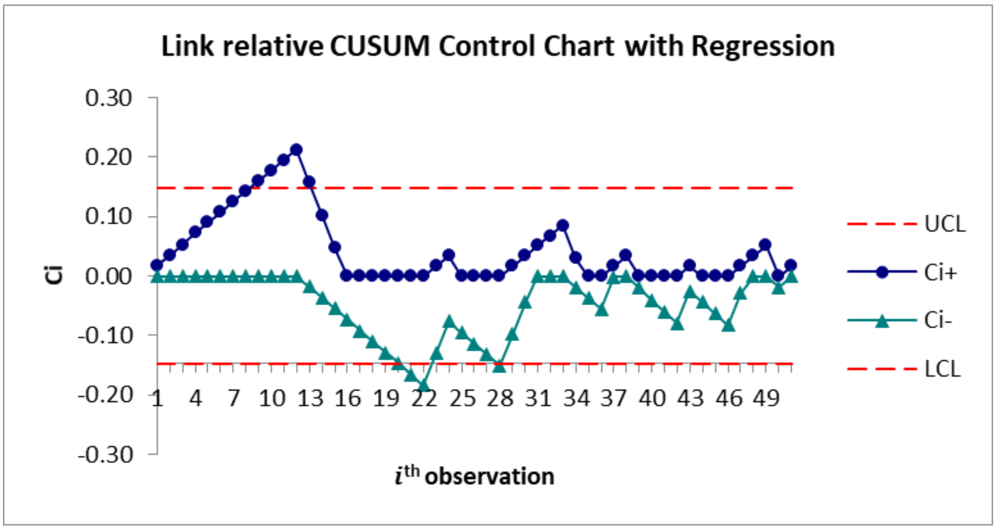

Fig. 5. Link Relative CUSUM Control Chart with Regression

Figure 5 shows that some data are still outside the control limits, there are 8 out of 51 data which are used as preliminary data. In this early stage a control chart will be determined in which all data is within the control limits. The Control Chart obtained next, can be used in monitoring further data. After 8 data is removed, a new custom control chart will be obtained as follows:

UCL = 0,147

LCL = -0,147

With a control chart and sample data as shown in Figure 6

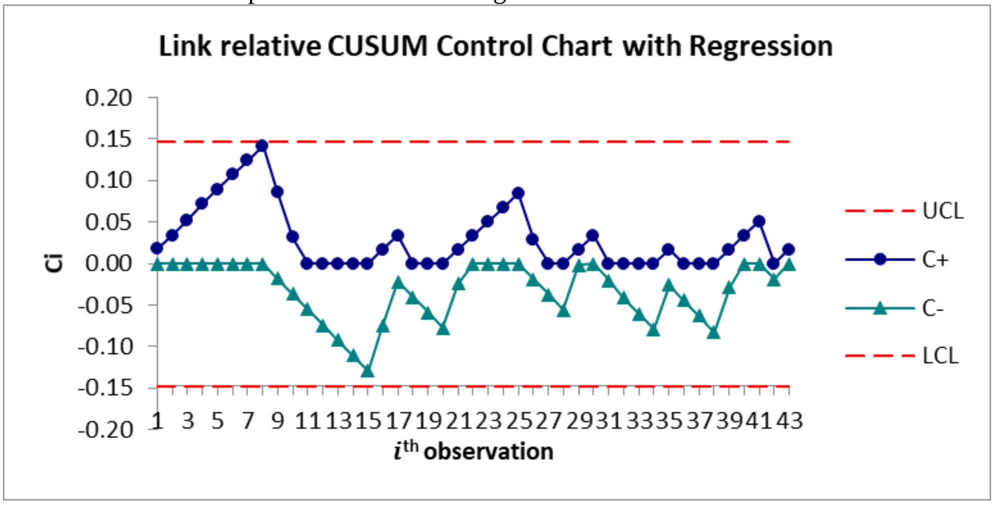

Fig. 6. Link Relative CUSUM Control Chart with Regression after the out-of-control data has been removed

In Figure 6, it can be seen that all data is within the control limits, so this control chart can be used to monitor further data. Control chart CUSUM, LR CUSUM and LR CUSUM Regression after all data outside the control limits is released the results show that the width of the CUSUM control chart is 0.32, the LR CUSUM control chart is 0.232 and the LR CUSUM Regression is 0.294. Thus the CUSUM regression control chart has a high sensitivity compared to the others.

4. Conclusion

Based on the results of data analysis that has been carried out using the three control charts, namely the CUSUM control chart, the Link Relative CUSUM Control Chart and the CUSUM Control Chart with the Regression approach, the CUSUM Regression LR control chart has a high sensitivity compared to the others. Thus, the control chart for the Feed Conversion Ratio (FCR) at SMK PPN Sembawa Palembang, Indonesia should use LCL -0.147 and UCL 0.147.

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