A Modified Exponentially Weighted Moving Average for Monitoring Zero Inflated Generalized Poisson Processes

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Abstract

Exponentially Weighted Moving Average (EWMA) control charts are commonly used to detect shifts in the process mean more quickly. The Zero-Inflated Generalized Poisson (ZIGP) distribution is a valuable method of statistical analysis for modeling data with excessive and overdispersion zero excees, particularly when analyzing counting or discrete data. In numerous cases, count data often have excessive number of zero outcomes than are expected in the Poisson. The ZIGP distribution can compensate for this overdispersion by including additional parameters that allow for greater than mean. This aim of this research is to adopt Modified-EWMA to monitoring the ZIGP processes as an alternative to handle overdispersion in zero-inflated count data. Parameter estimation ZIGP distribution using the maximum likelihood and Maximization Expectation algorithm with Newton Raphson approach. To improve the effectiveness of identifying tiny process mean shifts in overdispersion data, a control chart for the ZIGP distribution was devised in this study utilizing the modified EWMA statistic. The Average Run Length (ARL) of a Markov Chain with varied weighting constant values was used to evaluate the performance of the proposed control chart. Using the modified EWMA control chart, a superior detection ability was also compared to the old control chart. This was investigated using two sets of simulation data with different amounts of excess zero. The result shows that the ARL as a performance measure, the suggested ZIGP-Modified EWMA control chart is more suitable than the ZIGP-EWMA control chart.

Keywords: Overdispersion, Excess zero, ZIGP distribution, Modified EWMA, MLE, ARL, Markov Chain.

1. Introduction

In particular, most industrial processes are sensitive to small changes. A sudden change is a major change that occurs unexpectedly in a process that is monitored for a small shift. Based on Box (2016) EWMA is widely used in time series modeling and forecasting because EWMA can be seen as the weighted average of all observations and is not sensitive to normality assumption.

Many statistical quality control researchers have developed several methods to improve EWMA control chart detection capabilities for small to moderate shifts. Yaschin (1995) has discussed estimates of a process on small changes as well as sudden changes on the basis of EWMA. Capizzi & Masarotto (2012) have developed a new control chart of the EWMA control chart called the Adaptive-EWMA Control Chart to detect all types of shifts on data that are mutually independent. Reynolds & Stoumbos (2006) suggested using two EWMA control chart simultaneously, one for average shift processes and another to deal with variance shifts processes that detect small shifts with sudden changes. Shawky & Amal (1992) developed the Double EWMA control chart (DEWMA) that has been studied and used by many authors, such as Mahmoud & Woodall (2010) compared the EWMA control chart with the DEWMA, Kho et al. (2010) developed the Max-DEWMA control chart to detect small and moderate shifts, and Alevizakos & Koukouvinos (2021) studied the Modification EWBA control chart using a variety of additive parameter values and also proposed the Double Modified

EWMA (DMEWMA), to monitor the average shift of the process, assuming that the process follows the normal distribution.

Patel & Divecha (2011) introduced the EWMA Modification which considers previous observations and current process behavior in observations to detect early shifts in a production process effectively. Fandrilla et al. (2018) investigated the EWMA Modification control chart on automatically correlated data. Borror (1998) introduced the Poisson EWMA control chart with the assumption of distributed Poisson data. Aslam dkk. (2017) developed the EWMA Modification control chart to monitor the Conway Maxwell-Poisson process. Meanwhile, Mahmoud et al (2019) monitored a process with distributed Zero-Inflated Poisson data using an EWMA Adaptive Control Chart. Jamaluddin (2021) monitored data with the Zero Inflated Generalized Poisson (ZIGP) distribution and developed an EWMA control chart to address cases of overdispersion.

Overdispersion is a state in which the variation exceeds the mean. Overdispersion was discovered in data that contradicted the Poisson distribution's equispersion assumption. Generalized Poisson distribution (GP) can initially overcome cases of overdispersion. However, while the GP distribution can handle the problem of overdispersion, it cannot address the problem of inflated zeros or cases with a large number of zero values. As a result, a model that can deal with this issue is required. One distribution that can deal with the zero inflated problem is the Zero Inflated Poisson distribution (ZIP) Lambert (1992). ZIP distribution is a distribution that can be applied to data with more zero frequencies. Kusuma et al. (2013) applied a ZIP regression model to data overdispersion. However, this ZIP model is less suitable to address the problem of overdispersion and requires another suitable alternative model to solve the problem. One of these methods is the Zero Inflated Generalized Poisson Distribution Model (ZIGP). According to Famoye & Singh (2006), The ZIGP distribution is a combination of the ZIP and GP distributions and is an extension of the Poisson distribution. As a result, the ZIGP distribution can be used to spherical data that show the nature of overdispersion and have a higher number of zero frequencies.

Ni Wayan dkk (2019) applies ZIGP regression to overdispersion data. Maximum Likelihood Estimator (MLE) is one of the methods of measuring parameters that can be used to estimate the parameters of a model known distribution. The MLE method is used to maximize the likelihood function. Measurement of ZIP regression model parameters is done using the MLE method. Measurement of parameters with the MLE measuring method is calculated by maximizing its ln-probability function. The summing of ln-likelihood functions cannot be solved by ordinary numerical methods, therefore Expectation Maximization (EM) algorithms can be used.

The Average Run Length (ARL) measures the performance of the control chart to determine its sensitivity. Lucas and Saccucci (1990) used the Markov chain approach to calculate the run-length of an EWMA chart and published a computer program for calculating the average run-length for an EWMA scheme. Steiner et al. (1999) used an unhomogeneous Markov Chain to investigate the run-length nature of an EWMA control chart with a time-changing control border and suggested the Fast Initial Response (FIR) feature to detect quick shifts at the start of the process. Based on Khan et al. (2016) research, the creation of the modified EWMA control chart was more extensive and incorporated additional characteristics linked to errors between two observations with the assumption that the data is regularly or automatically correlated. Based on a preliminary literature study, the researchers wanted to create a modified EWMA control chart using ZIGP-distributed data and compare its performance outcomes to a traditional EWMA control chart using ZIGP-distributed data.

2. Materials and Methods

2.1. Zero-Inflated Generalized Poisson Distribution

The Zero-Inflated Generalized Poisson Distribution (ZIGP) is one of the distribution that can be used for data responses that are false. Famoye & Singh (2006) defined the ZIGP probability function as a combination of the ZIP and GP probability functions. The probability function of the ZIGP distribution can be written as follows:

$$f_{zigp}(x_i) = \begin{cases} \pi + (1-\pi)exp\left[\frac{-\lambda}{1+\omega\lambda}\right] & , x_i = 0 \\ (1-\pi)\left(\frac{\lambda}{1+\omega\lambda}\right)^{x_i} \frac{(1+\omega x_i)^{x_i-1}}{x_i!} exp\left[\frac{-\lambda(1+\omega x_i)}{1+\omega\lambda}\right] & , x_i > 0 \end{cases} \quad (1)$$

where λ is the mean of the underlying zero inflated generalized poisson distribution, ω is the parameter of dispersion, and π is the probability of data at $x_i = 0$.

Mean and variance of the ZIGP distribution are as follows:

$$E[X] = (1-\pi)\lambda \quad (2)$$
$$Var(X) = E[X][(1+\omega\lambda)^2 + \pi\lambda)] \quad (3)$$

2.2. Parameter Estimation of Zero Inflated Generalized Poisson Distribution

The maximum likelihood method is a method for estimating a parameter that maximizes a function of probability. If $n_1 + n_2 = n$ is the total of all observations that are assumed to be mutually independent, then the probability function is obtained from multiplying all its probability functions as follows:

$$L(\lambda,\pi,\omega|x_i) = \prod_{i=1}^{n_1} \pi + (1-\pi)exp\left[\frac{-\lambda}{1+\omega\lambda}\right]$$
$$\prod_{i=1}^{n_2}(1-\pi)\left(\frac{\lambda}{1+\omega\lambda}\right)^{x_i} \frac{(1+\omega x_i)^{x_i-1}}{x_i!} exp\left[\frac{-\lambda(1+\omega x_i)}{1+\omega\lambda}\right] \quad (4)$$

The log-likelihood function for the ZIGP distribution can be written as follows:

$$l_T(\lambda,\pi,\omega|x_i) = n_1 \ln\left(\pi_1 + (1-\pi_1)exp\left[\frac{-\lambda}{1+\omega\lambda}\right]\right) + n_2\ln(1-\pi) + \sum_{i=1}^{n_2} x_i \ln(\lambda) - \sum_{i=1}^{n_2} x_i \ln(1+\omega\lambda) +$$
$$\sum_{i=1}^{n_2}(x_i-1)\ln(1+\omega x_i) - \sum_{i=1}^{n_2}\ln(x_i!) - \sum_{i=1}^{n_2}\left[\frac{\lambda(1+\omega x_i)}{1+\omega\lambda}\right] \quad (5)$$

On the equation (5) it is not possible to know where the value zero is derived from the zero-state and which value zero comes from the poisson state so it is difficult to carry out calculations and make this function ln likelihood cannot be solved by the usual numerical method called also incomplete data likelihood. Therefore, to maximize the ln-likelihood function, the Expectation Maximation (EM) algorithm method is used so that the function of the likelihood is obtained as follows:

$$l_T(\lambda,\pi,\omega|x_i,z_i) = \begin{cases} \prod_{i=1}^{n_1} \pi^{z_i}\left((1-\pi)exp\left[\frac{-\lambda}{1+\omega\lambda}\right]\right)^{1-z_i} & , x_i = 0 \\ \prod_{i=1}^{n_2}\left((1-\pi)\left(\frac{\lambda}{1+\omega\lambda}\right)^{x_i}\frac{(1+\omega x_i)^{x_i-1}}{x_i!}exp\left[\frac{-\lambda(1+\omega x_i)}{1+\omega\lambda}\right]\right)^{1-z_i} & , x_i > 0 \end{cases} \quad (6)$$

Thus the total ln-likelihood function for the ZIGP distribution can be written as follows:

$$l_T(\lambda,\pi,\omega|x_i,z_i) = l_1(\lambda,\pi,\omega|x_i,z_i) + l_2(\lambda,\pi,\omega|x_i,z_i)$$
$$l_T(\lambda,\pi,\omega|x_i,z_i) = \sum_{i=1}^{n_1} z_i \ln\pi + \sum_{i=1}^{n_1}(1-z_i)\ln(1-\pi) - \sum_{i=1}^{n_1}(1-z_i)\left[\frac{\lambda}{1+\omega\lambda}\right] + \sum_{i=1}^{n_2}(1-z_i)\ln(1-\pi) +$$

$$\sum_{i=1}^{n_2} x_i(1-z_i)\ln(\lambda) - \sum_{i=1}^{n_2} x_i(1-z_i)\ln(1+\omega\lambda) + \sum_{i=1}^{n_2}(x_i-1)(1-z_i)\ln(1+\omega x_i) - \sum_{i=1}^{n_2}(1-z_i)\ln(x_i!) - \sum_{i=1}^{n_2}(1-z_i)[\lambda(1+\omega x_i)] - \sum_{i=1}^{n_2}(1-z_i)(1+\omega\lambda)$$

$$l_T(\lambda,\pi,\omega|x_i,z_i) = l(\pi|x_i,z_i) + l(\omega,\lambda|x_i,z_i) - \sum_{i=1}^{n_2}(1-z_i)\ln(x_i!) \quad (7)$$

2.3. Zero-Inflated Generalized Poisson – Exponentially Weighted Moving Average Control Chart

Z_t has concentration and spread values. The concentration value of an EWMA is a mean value while the spread value is a variance value. Suppose Z_t is defined as follows:

$$Z_t = \theta X_t + (1-\theta)Z_{t-1} \quad (8)$$

Mean of Z_t can be determined as follows:

$$E[Z_t] = E[\theta \sum_{k=0}^{t-1}(1-\theta)^k X_{t-k} + (1-\theta)^t Z_0] \quad (9)$$

Thus, Z_t will be influenced by $E(X_{t-k}), k = 0,1,2,\dots,t-1$. Based on the previous assumption $X_1, X_2, \dots, X_{t-1}$has an identical distribution, the ZIGP distribution. The mean and variance of ZIGP-EWMA is as follows:

$$E(Z_t) = (1–\pi)\lambda \quad (10)$$

$$Var(Z_t) = \frac{\theta(1-\pi)\lambda[(1+\omega\lambda)^2+\pi\lambda)]}{2-\theta}(1-(1-\theta)^{2t}) \quad (11)$$

By using development of the control charts of nonconformities based ZIGP distribution by Katamee (2013), (Katamee & Mayureesawan, 2013) the control limit is close to the stable value we have the UCL and LCL ZIGP-EWMA control chart, which is given by the following equation:

$$\begin{aligned} UCL &= (1-\pi)\lambda + L\sqrt{\frac{\theta(1-\pi)\lambda[(1+\omega\lambda)^2+\pi\lambda)]}{2-\theta}} \\ CL &= (1-\pi)\lambda \\ LCL &= (1-\pi)\lambda - L\sqrt{\frac{\theta(1-\pi)\lambda[(1+\omega\lambda)^2+\pi\lambda)]}{2-\theta}} \end{aligned} \quad (12)$$

2.4. Zero Inflated Generalized Poisson – Modified Exponentially Weighted Moving Average Control Chart

It is assumed that the quality characteristic of the interest denoted by X_i follows the ZIGP distribution with mean μ and variance σ^2. Based on this assumption, we propose the following control chart by generalizing the modified EWMA statistic intoduced by Patel & Divecha (2011). Suppose M_t is defined as follows:

$$M_t = \theta X_t + (1-\theta)M_{t-1} + k(X_t - X_{t-1}) \quad (13)$$

Where X_t is the sample mean at the time, k is a constant, and $0 < \theta < 1$ is a smoothing parameter.
The statistic Mt can also be expressed as

$$M_t = (\theta+k)X_t + \theta\sum_{j=1}^{t-1}(1-\theta-k)(1-\theta)^{t-j-1}X_j + (1-\theta-k)(1-\theta)^{t-1}\mu_0 \quad (14)$$

Mean of M_t can be determined as follows:

$$[M_t] = (\theta+k)\sum_{j=1}^{t}(1-\theta)^{t-j}(1–\pi)\lambda - k\sum_{j=1}^{t}(1-\theta)^{t-j}\,(1–\pi)\lambda + (1-\theta)^t E[M_o] \quad (15)$$

based on ZIGP distribution we know $E(M_0) = \mu_0 = (1{-}\pi)\lambda$, then:

$$E[M_t] = (\theta + k)(1{-}\pi)\lambda \sum_{j=1}^{t}(1-\theta)^{t-j} - k\,(1{-}\pi)\lambda \sum_{j=1}^{t}(1-\theta)^{t-j-1} + (1-\theta)^t(1{-}\pi) \qquad (16)$$

we obtained:

$$\begin{aligned} E[M_t] &= (\theta + k)(1{-}\pi)\lambda\left(\frac{1-(1-\theta)^t}{\theta}\right) - k\,(1{-}\pi)\lambda\left(\frac{1-(1-\theta)^t}{\theta}\right) + (1-\theta)^t\,(1{-}\pi)\lambda \\ &= (1{-}\pi)\lambda\left[(\theta + k)\left(\frac{1-(1-\theta)^t}{\theta}\right) - k\left(\frac{1-(1-\theta)^t}{\theta}\right) + (1-\theta)^t\right] \\ &= (1{-}\pi)\lambda[1-(1-\theta)^t + (1-\theta)^t] = (1{-}\pi)\lambda \end{aligned} \qquad (17)$$

and the variance value of the ZIGP-Modified EWMA is:

$$\begin{aligned} Var(M_t) &= \left\{(\theta + k)^2 + \theta^2(1-\theta-k)^2 \sum_{j=0}^{t-1}(1-\theta)^{2t-2j-2}\right\}Var(X_t) \\ &= \left\{\frac{(\theta+2\theta k+2k^2)-\theta(1-\theta-k)^2(1-\theta)^{2(t-1)}}{(2-\theta)}\right\}Var(X_t) \end{aligned} \qquad (18)$$

When the EWMA Modification control chart has been running for several periods, $(1-\theta)^{2(t-1)}$ to t becomes larger so that the control limit approaches the stable value given by the following equation:

$$\begin{aligned} Var(M_t) &= \frac{(\theta+2\theta k+2k^2)}{(2-\theta)}Var(X_t) \\ &= \frac{(\theta+2\theta k+2k^2)(1-\pi)\lambda[(1+\omega\lambda)^2+\pi\lambda)}{(2-\theta)} \end{aligned} \qquad (19)$$

After some simplification, based on mean and variance ZIGP we have the UCL and the LCL ZIGP-Modified EWMA control chart can be shown as:

$$\begin{aligned} UCL &= (1-\pi)\lambda + L\sqrt{\frac{(\theta+2\theta k+2k^2)(1-\pi)\lambda[(1+\omega\lambda)^2+\pi\lambda)}{2-\theta}} \\ CL &= (1-\pi)\lambda \\ LCL &= (1-\pi)\lambda - L\sqrt{\frac{(\theta+2\theta k+2k^2)(1-\pi)\lambda[(1+\omega\lambda)^2+\pi\lambda)]}{2-\theta}} \end{aligned} \qquad (20)$$

In the case of monitoring a process characterized by a nominant variable, a typical control chart includes a center line (CL) and lines that define the range of the typical variability of the analysed statistics: upper control limit (UCL) and lower control limit (LCL).

2.5. ARL for ZIGP-Modified EWMA Control Chart by Markov Chain Approach

Montgomery (2009) defines the Average Run Length (ARL) as the average number of plotted points on a control chart before a point signals an out-of-control state. As a result, the ARL is commonly used to report the control chart's success. If the process observations are independent, then the ARL may be calculated exactly for any Shewhart control chart using:

$$ARL = \frac{1}{P(out\ of\ control\ signal)} \qquad (21)$$

The Markov Chain Approach, originally proposed by Brook and Evans (1972) is an effective option for evaluating the ARL. It can also be applied to the EWMA control chart's ZIGP procedure to calculate the ARL. Areepong et al. (2014) studied this method, the interval between UCL and LCL values is divided into subintervals. The middle point m_i on the subinterval i^{th} can be written as follows:

$$m_i = LCL + \frac{(2i-1)(UCL-LCL)}{2N} \tag{22}$$

if statistic M_t is considered to be in the absorbing state if it does not fall within the control limits. As a result, the ARL is the expected time to absorption of the Markov chain. the ARL is the Markov chain's estimated time to absorption. The absorbing state, which is the out-of-control area below and above the control limits, is represented by the (N+1)th state,Q_{ij}, The one-step transition probability, Q_{ij}, is the probability of moving from state i at t-1 to state j at t at a certain point in time t, t = 1, 2,... This transition probability is denoted by:

$$Q_{ij} = P\left(L_j < M_t < U_j \,\middle|\, M_{t-1} = m_i\right) \tag{23}$$

$$Q_{ij} = P\left(\frac{LCL}{\delta} + \left(\frac{UCL-LCL}{2N\theta\delta}\right)\left(2j-(1-\theta)(2i-1)\right)\right) - P\left(\frac{LCL}{\delta} + \left(\frac{UCL-LCL}{2N\theta\delta}\right)\left(2(j-1)-(1-\theta)(2i-1)\right)\right)$$

Suppose **Q** contains the probabilities of going from one transient state to another, **I** is the N × N identity matrix, and **1** is a column vector of ones. The ARL based on t in-control states is given by (23), proved that

$$ARL = \boldsymbol{p}^T(\boldsymbol{1}-\boldsymbol{Q})^{-I}\boldsymbol{1} \tag{24}$$

where, $\boldsymbol{p}^T = (0, \dots, 0,1,0 \dots, 0)^T$ is initial state with 1 at the i^{th} coordinate and zeros elsewhere.

3. Results and Discussion

We examine the performance of control charts using two datasets: the first with 55% of zero and the second with 60% of zero. The Markov Chain in (23) and (24) provided the numerical results of the ARL of the ZIGP-EWMA control chart in (12) alongside those of the ZIGP-Modified EWMA control chart in (20). We assume that the observations are from the ZIGP distribution with parameters, and that the EWMA control chart's smoothing constant is $0 < \theta < 1$. For each dataset, the simulation is based on 500 single samples. We present the numerical results as shown in Table 1, Table 2, and Table 3.

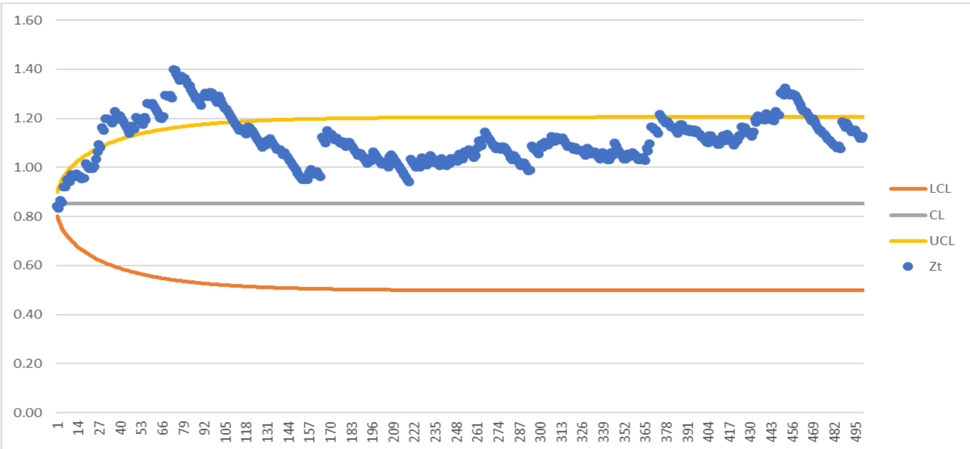

Fig. 1. ZIGP-EWMA Control Chart with $\theta = 0.01$ for first dataset

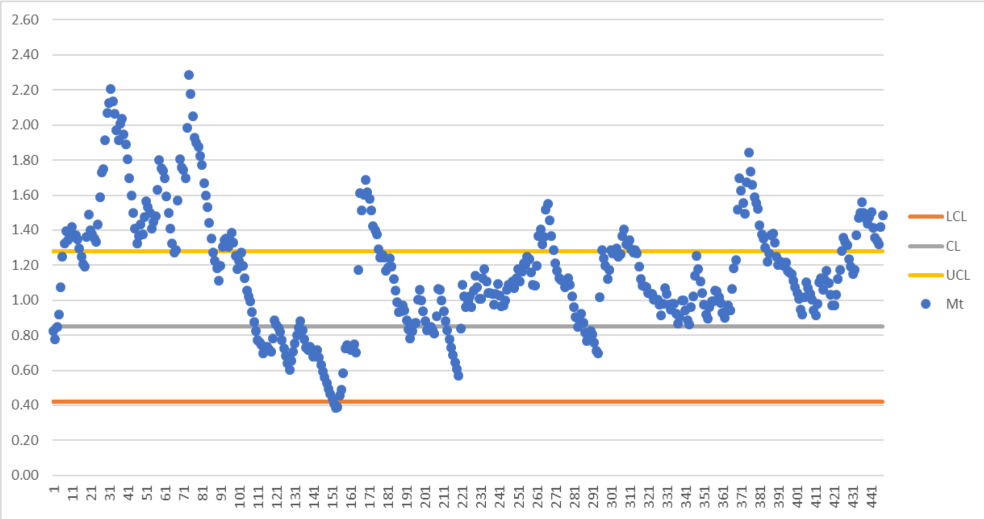

Fig. 2. ZIGP- Modified EWMA Control Chart with $\theta = 0.01$ for first dataset

Points indicating an out-of-control process can be found above or below the control limits. By plotting the statistics Z_t versus the sample number t, the ZIGP-Modified EWMA charts is created. If a plotted point Z_t lies over the UCL, the process is considered out of control. Otherwise, the process qualifies as in control and no changes in ZIGP parameters have occurred. A sequence of out-of-control signals are also noted.

Fig. 1. reveals that there are 110 out-of-control observations on the ZIGP-EWMA control chart, particularly data on observations 27, 29, 110, 440-441, and 447-466. Figure 2. Reveals that there are many points outside control limits. There is 142 out-of-control observations on the ZIGP-modified EWMA control chart, including data from the 6-15 observation, the 19-23 observation, the 25-43 observation, the 50-54 observation, the 56-63 observation, the 68-83 observation, and the 488-491 observation.

Table 1. Comparison ARL value of ZIGP-EWMA and ZIGP-Modified EWMA

No.	θ	ARL	
		ZIGP-EWMA	ZIGP-Modified EWMA
1	0.01	1.571528196	1.549996363
2	0.02	1.571342818	1.502700503
3	0.03	1.572985459	1.476691093
4	0.04	1.576991877	1.454747586
5	0.05	1.573985571	1.438064388
6	0.06	1.561563298	1.400962086
7	0.07	1.558430269	1.400962086
8	0.08	1.550625798	1.387927231
9	0.09	1.578272593	1.578272593
10	0.1	1.699464230	1.539595143
11	0.2	1.493260077	1.196658573
12	0.3	1.711548500	1.168325200
13	0.4	1.711548500	1.168325200
14	0.5	1.420434310	1.000000000
15	0.6	1.336650400	1.000000000

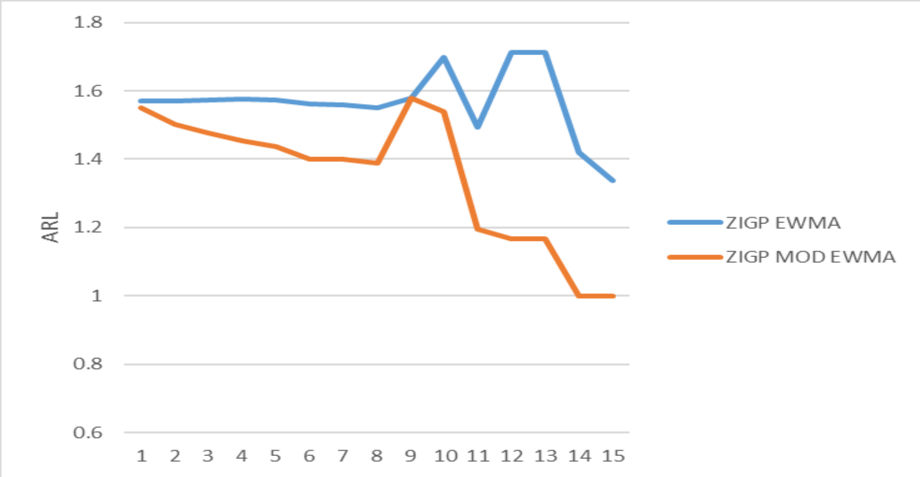

Fig. 3. illustrates the ARL value based on θ value changes for first dataset

Based on Table 1. the ARL value for each change value of θ shows the ARL value of the ZIGP-modified EWMA control chart, which is smaller than that of the ZIGP controlling chart. Although the ARL value for each control chart is the same under specific conditions as the one that occurs at = 0.09, the ARL value for both control charts is 1.578272593. Fig. 3. in general, any change in a larger θ value for each control chart reduces the ARL value, even if under certain conditions ARL values are increased. The ARL value obtained for the ZIGP-Modified EWMA control chart is smaller compared to the ARL values obtained for the ZIGP-EWMA control chart.

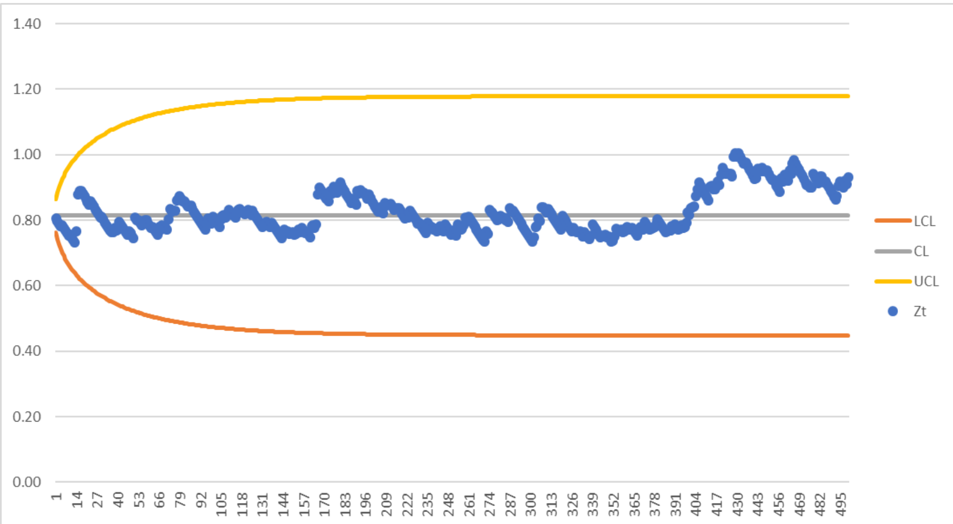

Fig. 4. ZIGP-EWMA Control Chart with $\theta = 0.01$ for second dataset

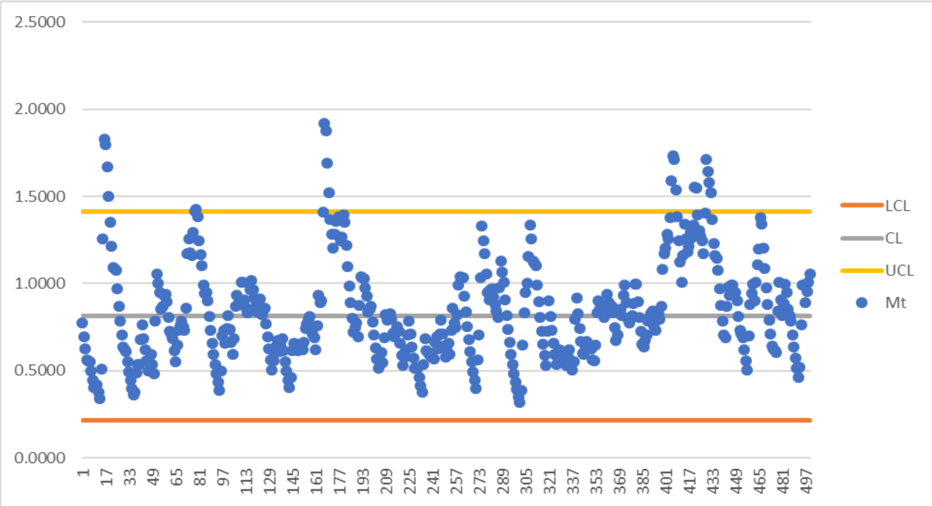

Fig. 5. ZIGP- Modified EWMA Control Chart with $\theta = 0.01$ for second dataset

Fig. 4. shows no observation data that is beyond the control limits on the ZIGP-EWMA control chart. Fig. 5. shows that there are 39 out-of-control observations on the ZIGP Modified EWMA control chart, namely data on the 15-19 observation, the 52-observation, 73-24, the 77-79, 274-276, 305,..., and the 495 observation. Therefore, it can be stated that the ZIGP-modified EWMA control chart is more sensitive in detecting outliers. By using the same method, the ARL value is obtained as follows:

Table 2. Comparison ARL value of ZIGP-EWMA and ZIGP-Modified EWMA

No.	THETA	ARL	
		ZIGP EWMA	ZIGP MOD EWMA
1	0.01	1.571628610	1.571528196
2	0.02	1.571680093	1.573193479
3	0.03	1.571758321	1.561024674
4	0.04	1.630527682	1.552733258
5	0.05	1.574249838	1.695686109
6	0.06	1.661826070	1.511024483
7	0.07	1.566639755	1.532907226
8	0.08	1.566923904	1.476691093
9	0.09	1.578272593	1.476691093
10	0.1	1.567862040	1.418784280
11	0.2	1.452565757	1.280442483
12	0.3	1.711548500	1.168325200
13	0.4	1.711548500	1.168325200
14	0.5	1.000000000	1.000000000
15	0.6	1.000000000	1.000000000

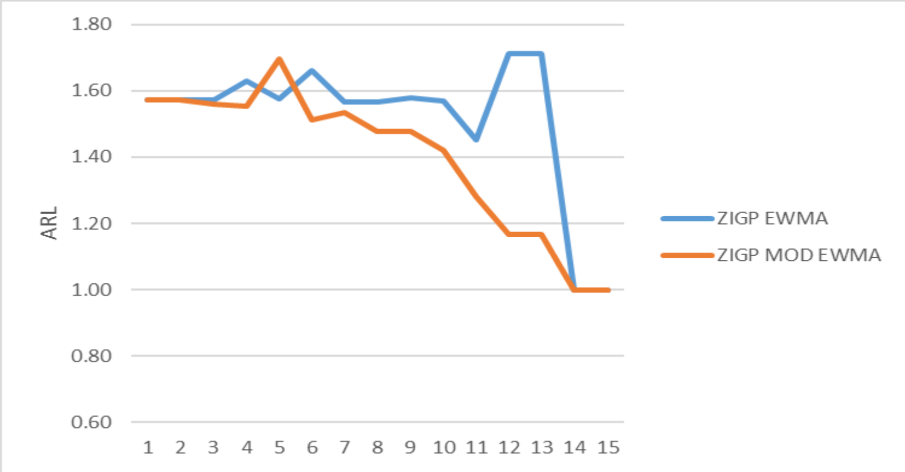

Fig. 6. ARL value based on θ value changes for second dataset

According to Fig. 6. every change in a larger θ value for each control chart reduces the ARL value, even if the ARL value is increased under specific conditions. The ARL value for the ZIGP-Modified EWMA control chart is lower than the ARL value for the ZIGP-EWMA control chart. To see the sensitivity of the ZIGP-modified EWMA control chart for outlier detection based on zero data frequency, see the following **table 3.** below:

Table 3. ARL values based on the frequency of zero values on data

NO.	THETA	ZIGP-Modified EWMA	
		0.55	0.60
1	0.01	1.549996363	1.153417165
2	0.02	1.502700503	1.535263727
3	0.03	1.476691093	1.571758321
4	0.04	1.454747586	1.438064388
5	0.05	1.438064388	1.400962086
6	0.06	1.400962086	1.400962086
7	0.07	1.400962086	1.424264750
8	0.08	1.387927231	1.323523559
9	0.09	1.578272593	1.604011489
10	0.1	1.539595143	1.418784280
11	0.2	1.196658573	1.280442483
12	0.3	1.168325200	1.168325200
13	0.4	1.168325200	1.168325200
14	0.5	1.000000000	1.000000000
15	0.6	1.000000000	1.000000000

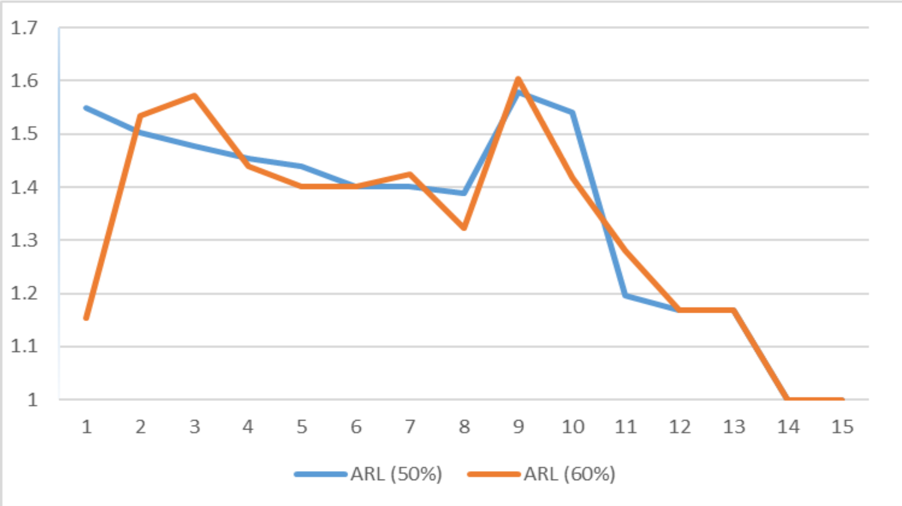

Fig. 7. ARL value based on persentage of zero data for every changes value of θ

Based on Fig. 7. ARL values for changes in θ values on the ZIGP-modified EWMA control chart are smaller on data with larger zero data frequencies. Although some ARL values on data with zero data frequency are 60% larger than data with 55% zero data, under certain conditions the value of ARL increases as it occurs at θ=0.02, 0.03,0.07,0.09 and 0.2 so the ZIGP-EWMA will be the preferred control chart.

4. Conclusions

We proposed a Zero Inflated Generalized Poisson–Modified Exponentially Weighted Moving Average for process monitoring and control for overdispersion and excess zero data. Based on research, the known ARL value of the ZIGP-modified EWMA control chart is smaller than the ARL values of the zigp-ewma control chart, so it can be said that the ZIGP-modified EWMA control chart is better used on data that are overdispersed and excess zero or on data that has a higher frequency of zero, so the ability of the ZIGP-modified EWMA control chart to detect signals out of control performs is better than using the ZIGP-EWMAcontrol chart.

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