

# Planning the trajectory of the 3RRR manipulator

**Bourama Coulibaly<sup>1</sup>, Marie Bernard Sidibé<sup>2</sup>, Kya Abraham Berthé<sup>3</sup>**

1. Research and Training Center for the Textile Industry (CERFITEX), Department of Electricity, Electronics and Automation (EEA) Ségou, Mali.  
bcoulibaly28@yahoo.fr
2. National School of Engineers, Department of Industrial Engineering, Bamako, Mali.  
sidibebernard@yahoo.fr
3. University of Technical and Technological Sciences, Faculty of Technical Sciences, Physical Department, Bamako, Mali.  
berthemoni@yahoo.fr

**Abstrat :** *The manipulator provides a gain in terms of production rate compared to conventional serial manipulators. The trajectory planning made it possible to optimize the performance of the parallel 3RRR manipulator in its workspace. This planning was done on the octave software, based on the modeling of the manipulator using the inverse geometry method. Which allowed to determine the workspace of the robot. The working space of a parallel robot therefore depends on the different independent kinematic chains that connect the base to the effector (mobile platform).*

**Key words:** 3RRR parallel manipulator, trajectory planning, modeling, independent kinematic chains, Works pace.

## I. Introduction

In recent years, parallel robots have received considerable attention for various research and applications. Parallel manipulators are closed-loop mechanisms in which separate serial chains (links and joints) are connected to both the fixed base and the mobile platform.

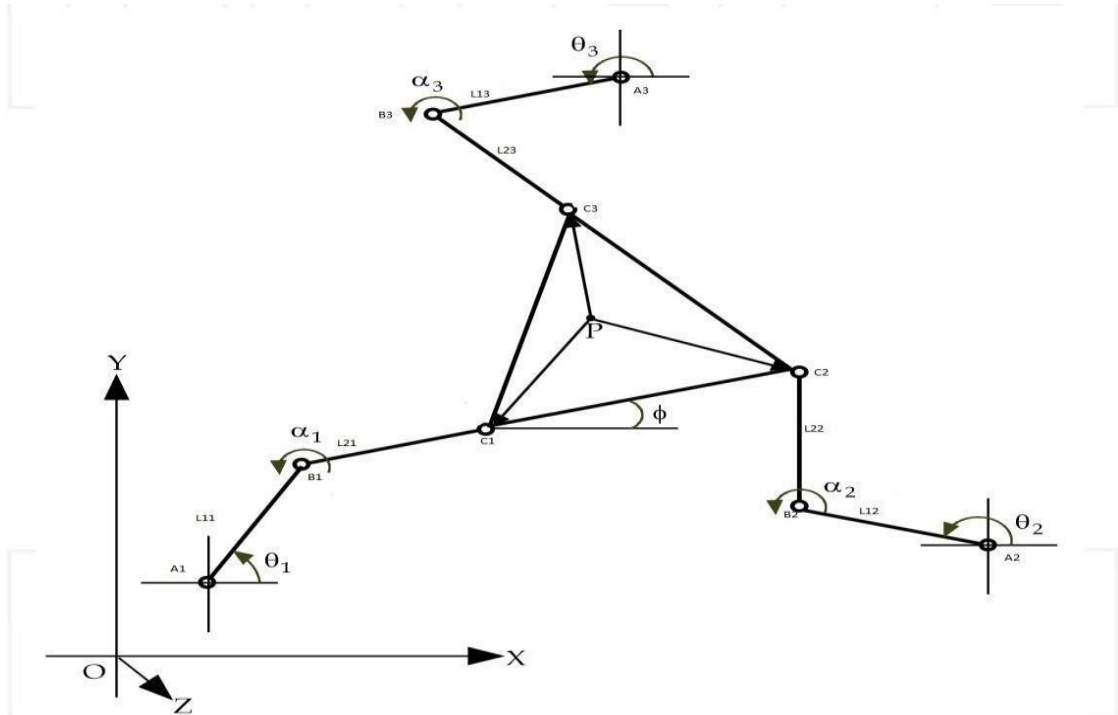


Figure 1: The geometric representation of the parallel robot 3RRR.

Source: Nantes Institute for Research in Communications and Cybernetics / 2010

The angles  $\alpha_i$  and  $\theta_i$  define the angular position of the segments of the leg  $i$ ,  $i \in [1, 2, 3]$ , which depend on the relative position of the anchor points  $A_i$ ,  $B_i$ , and  $C_i$ , illustrated in *figure1*. In order to simplify the resolution, one places oneself in the fixed landmark of origin  $O$ . The axis of this landmark is carried by the axis of rotation of the center pivot link  $A_i$ . The Cartesian coordinates of the points  $C_i$  are known because these points are related to the effector whose orientation is known, but is expressed in the reference  $(x, y, z)$  of the origin  $C_1$  illustrated in FIG. It is necessary to express the coordinates  $C_i$  in this reference, which is obtained by a rotation of angle  $\Phi$  around the axis  $x$  [3].

This article highlights the various possible movements of the manipulator in order to predict or plan its trajectory based on a modeling of the manipulator by the inverse geometry method.

## II. Parallel 3RRR manipulator architecture

The minimal architecture for a parallel 3RRR robot requires three actuators to drive the three main rotations in the Cartesian space illustrated in Figure 2. Important data for a robot is the expression of the values that must be taken by the controlled actuators to reach the desired configuration. [2]

This information is called the Geometric Problem, it gives the link between the quantities controlled by the actuators called joint coordinates and the state variables of the robot that we want to control called Cartesian coordinates.

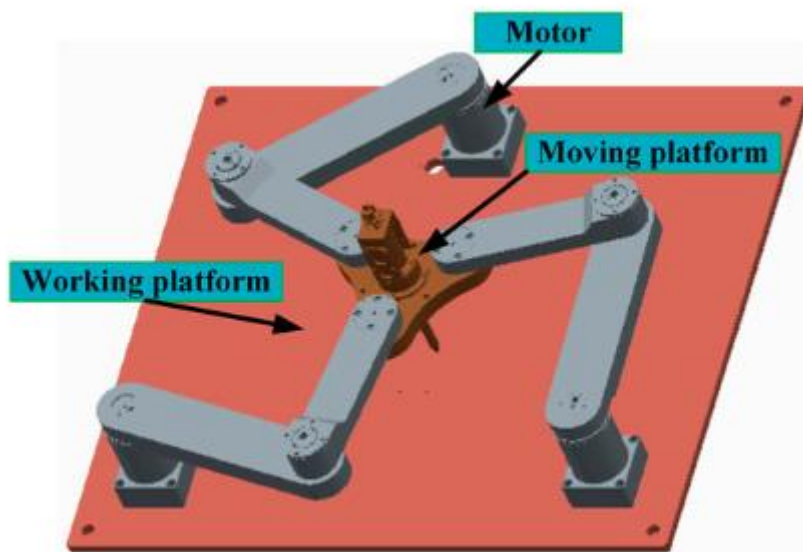


Figure 2: Architecture of a Parallel 3RRR Robot.

Source: Article by Lia chao Sheng and Wei Li (Optimization Design by Genetic Algorithm Controller for Trajectory Control of a 3-RRR Parallel Robot)

## III. Modeling the parallel 3RRR robot by inverse geometry

This method is based on the Denavit and Hartenberg convention (DH 1955).

It allows the passage between adjacent joints of a robotic system. It concerns open kinematic chains where the joint has only one degree of freedom, and the adjacent surfaces remain in contact. For this aspect the use of hinges or slides is essential. The appropriate choice of reference points in the links

facilitates the calculation of the matrices and makes it possible to quickly express information from the terminal element to the base or vice versa. [4]

From Figure 1 we can write the following equation:

$$OA_i + A_i C_i = OP + PC_i \quad (1)$$

For  $i = 1, 2$  and  $3$ , we can write the following two matrices of transformations:

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & o_{xi} \\ 0 & 1 & 0 & o_{yi} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta_i & -\sin \theta_i & 0 & 0 \\ \sin \theta_i & \cos \theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha_i & -\sin \alpha_i & 0 & L_{1i} \\ \sin \alpha_i & \cos \alpha_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L_{2i} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_i + \alpha_i) & -\sin(\theta_i + \alpha_i) & 0 & o_{xi} + L_{2i}\cos(\theta_i + \alpha_i) + L_{1i}\cos\theta_i \\ \sin(\theta_i + \alpha_i) & \cos(\theta_i + \alpha_i) & 0 & o_{yi} + L_{2i}\sin(\theta_i + \alpha_i) + L_{1i}\sin\theta_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & P_{xp} \\ 0 & 1 & 0 & P_{yp} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\gamma_i + \phi) & -\sin(\gamma_i + \phi) & 0 & 0 \\ \sin(\gamma_i + \phi) & \cos(\gamma_i + \phi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & n_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

The Cartesian coordinates P (PXF, PYF) correspond to the position of the effector in the fixed reference. In addition, the two matrices are equal, so the equation becomes:

$$\begin{bmatrix} L_{2i}\cos(\theta_i + \alpha_i) \\ L_{2i}\sin(\theta_i + \alpha_i) \end{bmatrix} = \begin{bmatrix} P_{xp} + b_{xi}\cos\phi - o_{xi} - L_{1i}\cos\theta_i \\ P_{yp} + b_{xi}\sin\phi + b_{yi}\cos\phi - o_{yi} - L_{1i}\sin\theta_i \end{bmatrix} \quad (4)$$

With  $b_{xi} = n_i \cos \gamma_i$  and  $b_{yi} = n_i \sin \gamma_i$ . By squaring the two lines of equation (4) we obtain easily after simplification the following equation:

$$L_{2i}^2 - 2P_{yp} o_{yi} - 2P_{xp} o_{xi} + b_{2i}^2 X_i + b_{2i}^2 Y_i + o_{2i}^2 X_i + o_{2i}^2 Y_i + P_{2i}^2 X_p + P_{2i}^2 Y_p + 2L_{1i} b_{yi} [\sin(\phi - \theta_i) - \cos(\phi - \theta_i)] + 2\cos\phi (P_{xp} b_{xi} + P_{yp} b_{yi} - b_{xi} o_{xi} - b_{yi} o_{yi}) + 2\sin(\phi) (P_{yp} b_{xi} - P_{xp} b_{yi} - b_{yi} o_{yi} + b_{xi} o_{xi}) + 2L_{1i} \cdot \cos\theta_i (o_{xi} - P_{xp}) + 2L_{1i} \cdot \sin\theta_i (o_{yi} - P_{yp}) - L_{2i}^2 = 0 \quad (5)$$

For the calculation of the inverse geometric model, equation 5 can be written in the form:

$$A_i \sin \theta_i + B_i \cos \theta_i = C_i$$

With :

$$A_i = 2L_{1i} (o_{yi} - b_{xi} \sin(\phi) - b_{yi} \cos(\phi) + P_{yp})$$

$$B_i = 2L_{1i} (o_{xi} - b_{yi} \sin(\phi) - b_{xi} \cos(\phi) + P_{Xp})$$

$$C_i = -[L_{21i} - 2P_{Yp} o_{yi} - 2P_{Xp} o_{xi} + b_{2Xi} + b_{2Yi} + o_{2xi} + o_{2yi} + P_{2Xp} + P_{2Yp} - L_{22i} + 2 \cos \phi (P_{Xp} b_{xi} + P_{Yp} b_{yi} - b_{xi} o_{xi} - b_{yi} o_{yi}) + 2 \sin(\phi) (P_{Yp} b_{xi} - P_{Xp} b_{yi} - b_{yi} o_{yi} + b_{xi} o_{xi})]$$

$$\Theta_i = \text{Atan2}(A_i, B_i) \pm \text{Atan2}(\sqrt{A_i^2 + B_i^2 + C_i^2}, C_i) \quad (7)$$

Equation 7 presents two possible solutions for each arm so we have eight (8) possible solutions for the 3RRR robot.

The rotation of active and passive joints is obtained by theoretical equations above. So we can go to the simulation on the octave software.

## VI. Simulation

This program includes several MATLAB scripts used for the simulation of 3-RRR parallel manipulators. The program consists of interactively defining the position and orientation of the mobile platform. At each change of pose, the inverse kinematic problem is solved and the new configuration is drawn. In addition, at each change of orientation of the mobile platform, the work space of constant orientation is calculated and plotted.

### 1. 3RRR Robot Settings

Arm length:

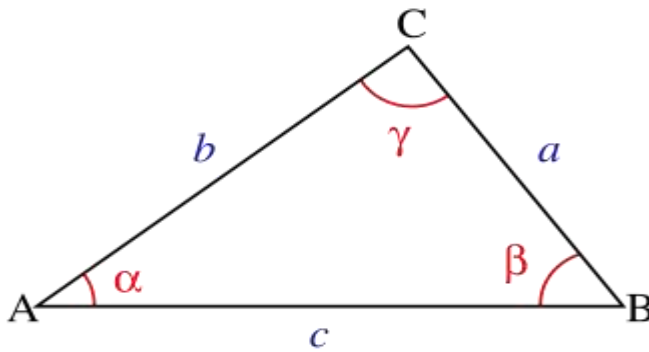
$$l_{1i} = 7.2 \text{ cm } l_{2i} = 9.6 \text{ cm}$$

For the calculation of triangle parameters, we used the sinus law [2]:

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

And the cosine law:

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$



### 2. Origin coordinate

$$o_{x1} = 0 \text{ cm}, o_{x2} = 24.3 \text{ cm}, o_{x3} = 1.903510807 \text{ cm}; o_{y1} = 0 \text{ cm}, o_{y2} = 0 \text{ cm}, o_{y3} = 24.09994339 \text{ cm},$$

### 3. Parameter of the triangle

$n = 5.820652885\text{cm}$ ,  $n = 5.820652885\text{cm}$ ,  $n = 3.810511777\text{ cm}$ ;  $\gamma_1 = 220.8933947\text{ degree}$ ,  $\gamma_2 = -40.933947\text{ degree}$ ,  $\gamma_3 = 90\text{ degree}$ ;  $\Phi = 0\text{ degree}$ ;

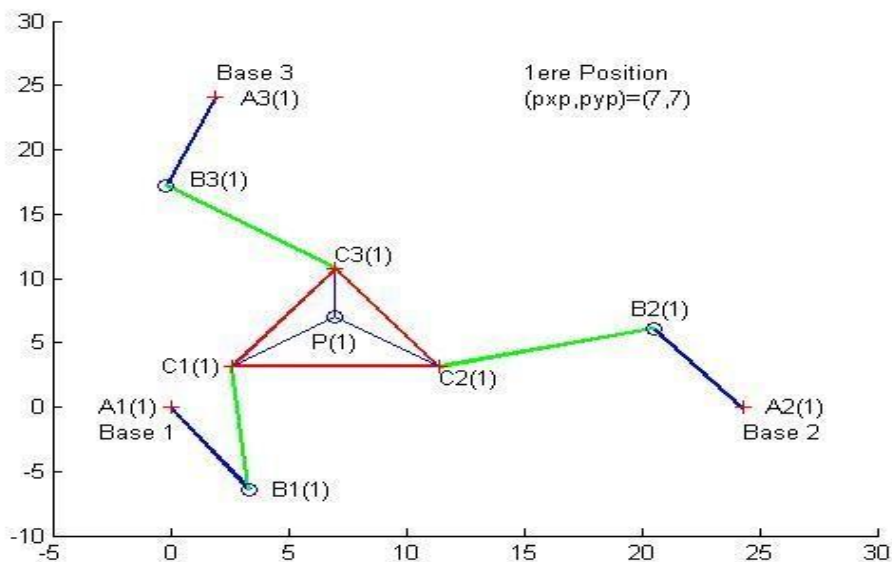
#### 4. The walk of the parallel 3RRR Manipulator

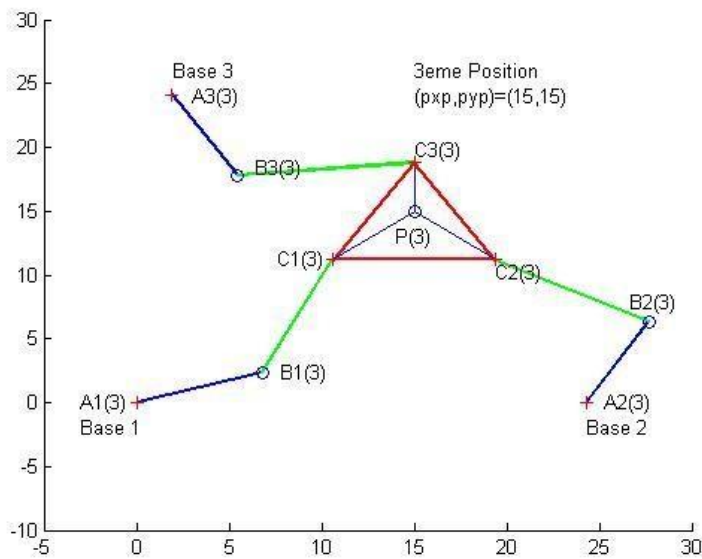
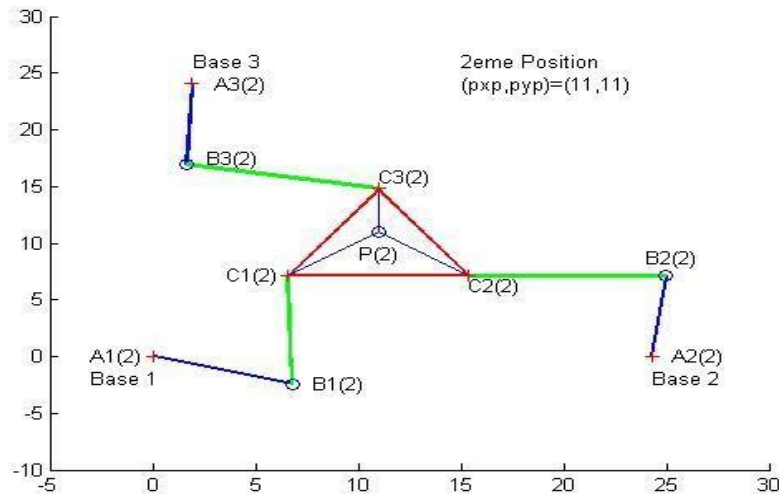
Thus, the course of the parallel 3RRR manipulator according to cartesian coordinates of the effector or the mobile platform is determined. The range of possible movements of the manipulator is between the two cartesian coordinates of the effector (departure threshold and arrival threshold) of the table below.

**TableI:** The possible movements of the manipulator.

Cordonné of Departure	Cordonnee of arrival
(6 ,5)	(16,15)

In *TableI* above, which represents the set of possible paths of the mobile platform or the effector between the two Cartesian coordinates P1 (6, 5) and P2 (16,15). Between these two coordinates, there are several possible movements of the manipulator. For example, a simulation of the three movements of the mobile platform of the robot is shown below:





## 5. The working space of each arm of the parallel robot

Using numerical simulations, the complete workspace is identified for each arm of the manipulator shown above.

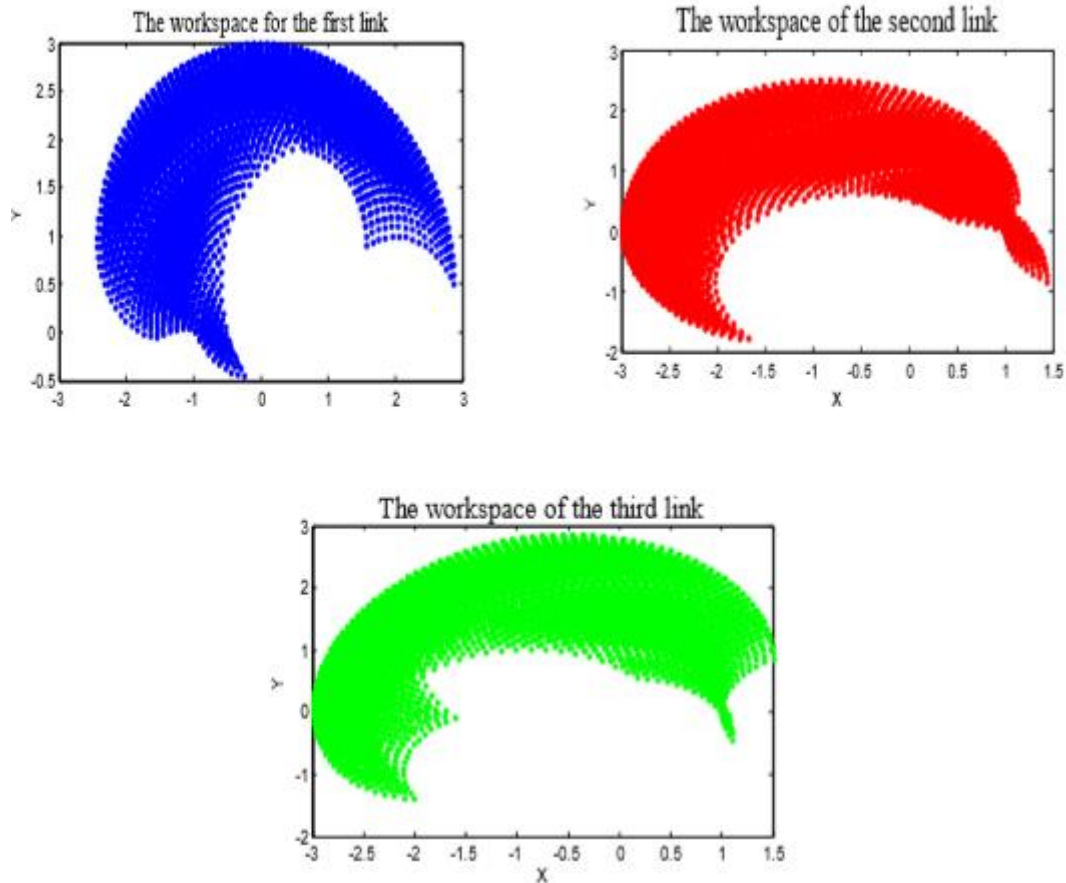


Fig.3: the workspace of each link of the manipulator

## 6. The manipulator's working space

This is the total workspace obtained by intersecting three regions of the robot's legs

Thus, the workspace of the parallel 3RRR manipulator is summarized in the following figure:

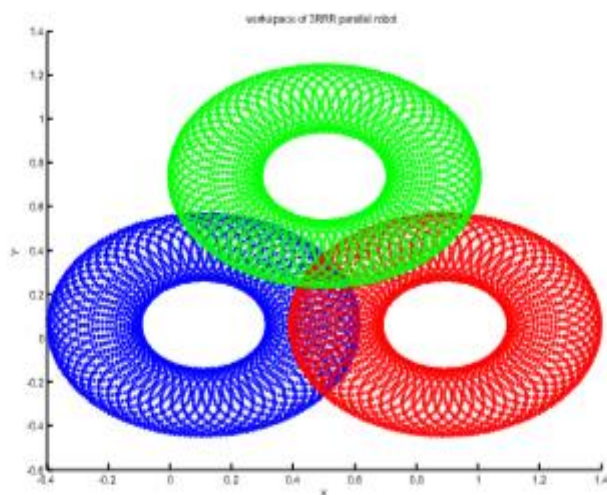


Fig 3: Intersection between the three regions of mobility of the robot

## V. Conclusion

The planning of the 3RRR parallel manipulator workspace depends on the working space of each arm of the manipulator. The analysis of the robot's work space led to a high performance and a better positioning of the platform in order to optimize the trajectory of the manipulator. A modeling of the manipulator by the inverse geometry method was presented to determine the inverse kinematics.

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